EINDHOVEN UNIVERSITY OF TECHNOLOGY

2MMR10 - MODELLING WEEK

Modelling the Sharpe ratio for investment strategies

Group 6

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December 23, 2016

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1 Introduction

Investing is the act of committing money or another certain type of capital to an endeavor with the expectation of retrieving a higher amount of money. The goal of investing is to maximize the expected return while minimizing the risk of losing the invested capital. There are numerous asset classes in which an investor can invest. The five most important categories are stocks, bonds, gold, real estate and cash. Each of these categories has its own risks and historical returns, these risks and historical returns are correlated with each other. To lower the risks investors invests in different asset classes. This is called asset allocation.
New High invests in a mechanical way. They have a predetermined asset allocation strategy which they use to perform a strategy that consults you when to sell or buy different assets to return to the predetermined asset allocation. The base strategy is called the Permanent Portfolio strategy. This strategy is first described in 1980 by Harry Browne. The predetermined asset allocation of the permanent portfolio strategy is the total capital equally invested in stocks, bonds, gold and cash. The value of the different assets changes with regard to time. When one of the assets is represented for more than 30% or less than 20% in the portfolio the Permanent Portfolio strategy automatically rebalances every asset to 25% by buying or selling these assets.
The performance of the investment strategy can be measured by the corresponding Sharpe ratio. The Sharpe ratio takes the expected return of the method, the expected risk free rate and the standard deviation of the return of the method into account,

$$SR := \frac{E\left[R - R_f\right]}{\sigma}.$$

The Sharpe ratio should at least be higher than zero in order for the strategy to be of any value. A Sharpe ratio greater than 0.5 is considered good and a Sharpe ratio higher than 1.0 is considered to be very good. As a reference, the base Permanent Portfolio strategy has a Sharpe ratio of 0.78. The challenge is to come up with a strategy that performs better than the base strategy. New Highs has an idea for a different strategy. This idea involves including another asset, namely the volatility asset category, tuning the allocation percentages and introducing a single trading rule, namely: if the number of stocks on the New York Stock Exchange that make new 1-year highs is greater than the number of stocks that make 1-year lows then switch to a different allocation scheme. New Highs has made software that optimized this strategy based on data from the past. The outcome of this strategy yields a high Sharpe ratio. New Highs wants to know whether their new strategy performs as well in the future as it would have done in the past.
In order to solve this question a model is written based on the past data. With this model different data sets are made which are all possibilities for the future. With the use of Monte-Carlo simulation the Permanent Portfolio strategies is tested on these data sets. This can possibly give insight on how the strategies perform in the future. In the remainder of this report the model and the mathematical explanation is provided. The results of the Monte-Carlo simulation are provided and discussed. Finally an idea on how to use our findings for further research is presented.

2 Mathematical model

The mathematical model consists of two different parts. The first part models the historical data for cash, bonds, stocks and gold and the modelling is explained in the first subsection. The second part performs Monte Carlo simulation based on the data that is computed by the model described in the first part.

2.1 Modelling the historical data

For now, it is assumed that there are four asset categories, namely: cash (C), bonds (B), stocks (S) and gold (G). Each asset category follows a seperate time series. Not all the asset categories have the same starting dates for the sample of their time series, because trading in some assets started later than others or because the data is not available. Therefore the largest coverage of available data is taken. This is in our case at the last day of each month from March 2004 till November 2016. Now that the data has equal length the data is ready to be prepared. First the logarithm of the values for every data set is taken, because by testing we saw that the time series are multiplicative. This assumption of multiplicativity was taken for granted after multiple tests on the datasets. To model the time series we need a starting point of 0. this is why afterwards the first element of each data set is subtracted from every row of that data set. Subsequently the slope is calculated, this is the average trend per time step. This is a very coarse manner to estimate the trends in the several asset categories. One can observe that for the four categories an overall growth is present, therefor only an overall trend is incorporated in the following model for simplicity reasons. Where the average trend is computed by taking the overall trend divided by the number of time steps. Then the slope multiplied by the current time step is subtracted from the data sets to obtain four stationary series.

The time series are correlated. A fairly easy way to this is to model each time series as the simplest auto-regressive model that takes correlations into account, the VAR(1) model. vector auto-regression (VAR) models the analysis of multivariate time series. The VAR model are useful for describing the dynamic behavior of economic time series and for forecasting. In the VAR(1) model, each time series follows an Auto-regressive model (AR(1)) model. An AR(1) auto-regressive model is defined by

$$X_t = c + \phi X_{t-1} + \varepsilon_t, \tag{1}$$

where X_t is the value of the time series at moment t (in our case the last day of the month) depending on the value X_{t-1} of the previous timestep and some constants, c is the constant growth per time unit, ϕ is the model parameter to indicate the importance of the previous value X_{t-1}, and ε_t is a normally distributed random variable with mean 0. Thus, a value X_t depends solely on the previous value and a random value. [2]

For the four assets the combined time series is given by:

$$X_{C,t} = c_C + \phi_C X_{C,t-1} + \varepsilon_{C,t} \tag{2}$$

$$X_{B,t} = c_B + \phi_B X_{B,t-1} + \varepsilon_{B,t} \tag{3}$$

$$X_{S,t} = c_S + \phi_S X_{S,t-1} + \varepsilon_{S,t} \tag{4}$$

$$X_{G,t} = c_S + \phi_G X_{G,t-1} + \varepsilon_{G,t}. \tag{5}$$

Note that c_C, c_B, c_S and c_G, correspond to the slope that was extracted earlier on, so these

values do not need to be considered in the rest of the model.
The parameters ϕ_C, ϕ_B, ϕ_S and ϕ_G are estimated by maximum likelihood estimation.
These equations could be written as vectors, giving $Y_t = AY_{t-1} + U_t$, or in full:

$$\underbrace{\begin{bmatrix} X_{C,t} \\ X_{B,t} \\ X_{S,t} \\ X_{G,t} \end{bmatrix}}_{Y_t} = \underbrace{\begin{bmatrix} \phi_C & 0 & 0 & 0 \\ 0 & \phi_B & 0 & 0 \\ 0 & 0 & \phi_S & 0 \\ 0 & 0 & 0 & \phi_G \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} X_{C,t-1} \\ X_{B,t-1} \\ X_{S,t-1} \\ X_{G,t-1} \end{bmatrix}}_{Y_{t-1}} + \underbrace{\begin{bmatrix} \varepsilon_{C,t} \\ \varepsilon_{B,t} \\ \varepsilon_{S,t} \\ \varepsilon_{G,t} \end{bmatrix}}_{U_t} \tag{6}$$

To complete the model, the covariance matrix of U_t needs to be known.
If Σ denotes the covariance matrix of Y_t mentioned above and Q denotes the covariance matrix of U_t then one obtains:

$$\Sigma = A\Sigma A^T + Q \tag{7}$$

By the use of the vectorization (vec) of a matrix and the Kronecker product ($\otimes$) one is able to calculate the values of Q:

$$\text{vec}(Q) = [I - A \otimes A^T]\,\text{vec}(\Sigma). \tag{8}$$

Σ can be estimated from the data, so, now the covariance for the white noise terms can be estimated. These terms are generated from a multivariate normal distribution with means 0 and the given covariances. Now all ingredients for the model are available and thus a time series for the four asset categories can be generated. By inverting the first steps the generated terms become suitable for the simulations of the possible futures.

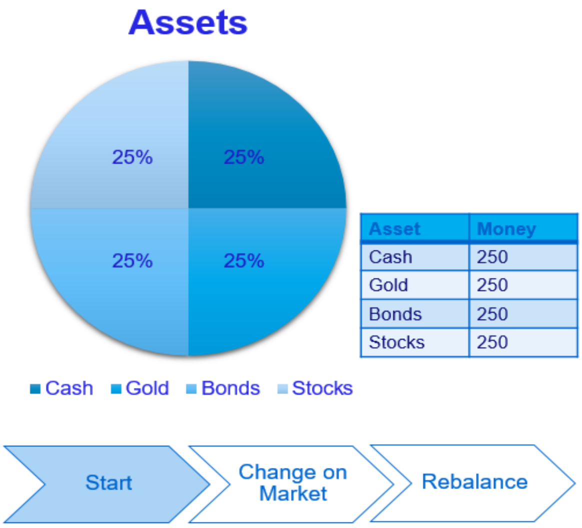
2.2 Strategies and Monte Carlo simulations

First of all, there is need for a strategy to analyze the given data. The only considered strategy here is the Permanent Portfolio (PP).

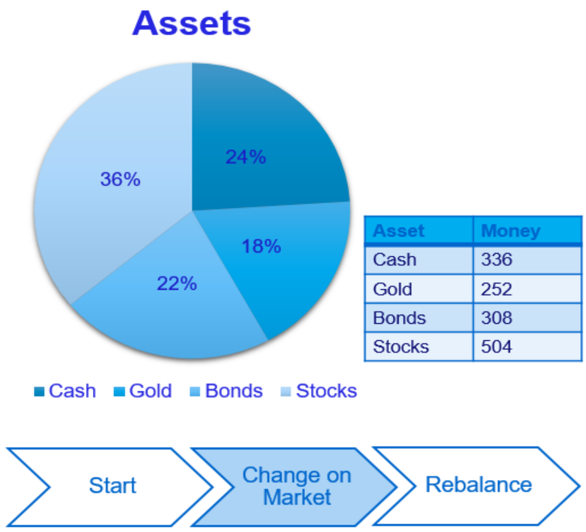
The idea of the Permanent Portfolio was published in 1999 by Harry Browne, the Permanent Portfolio is a strategy to invest capital risk free, no matter what happens in the future. [1]

The strategy of Permanent Portfolio is based on the the fact that in each economic cycle one of the assets performs well. We will describe the Permanent Portfolio according the first idea (classic Permanent Portfolio) to keep it simple. Suppose we have 1000 euros and want to invest in the categories: cash, gold, bonds and stocks. We put 25% of our money in each of the four assets. This is the default situation. If after some changes on the economic market, an asset differs more than a certain percentage from the default of 25% then we redivide all the money to go back to the default situation. This is called re-balancing. After some time the value of the different assets change and thus our money that we have in the assets will have shifted. The new percentages of the total can be calculated. Suppose we have a situation which has 24% cash, 18% gold, 22% bonds and 36% stocks. Assume the maximum difference to the default percentage is 20%. Then it would mean that all of the four assets are within this bound and we are not going to re-balance. If this maximum difference is only 10% then we are going to re-balance. Re-balancing is done by calculating the total investments of that moment and dividing it equally across all the assets.

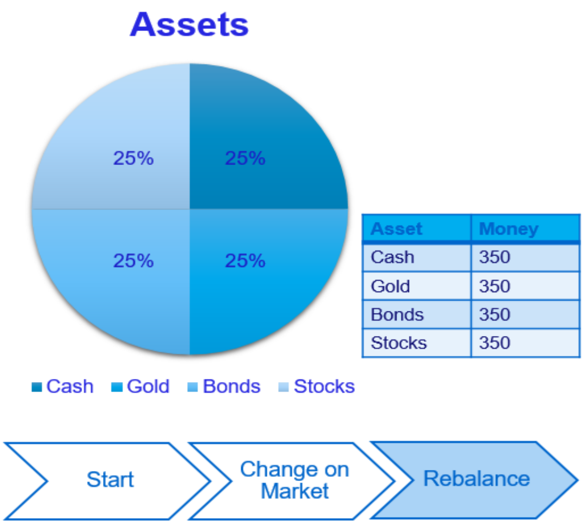
In Figure 1 an example of the process can be seen.



(a) Default state



(b) Change on market



(c) Rebalance

Figure 1: Example of the reaction of classical PP on a change on the market.

Above the classical Permanent Portfolio is discussed. However a lot of variations are made on this strategy. The number of assets used can be changed. For example if an investor does not trust investing in stocks he can switch these out for another asset. However, the strength of the Permanent Portfolio strategy is that in each state of the economy there is at least one asset performing well. If this is not the case with the variation it could be the case that the strategy does not have a positive return on investment.

Besides changing the number of assets, it is also possible to change the target percentages per asset. For example in the default situation we could choose to have 40% gold and the other three 20%. The last parameter that can be changed is the maximal difference between the current percentage and the target percentage.

The time series for all four asset categories are first modelled by the historical data. Then the simulation will start with the last known value and will be updated iteratively every next time step. For simulation of the time series, multivariate normal distributed variables with mean 0 and the covariance matrix from the previous subsection are drawn. These are the innovations for each time series. After the time series of the four asset categories are generated one can apply the Permanent Portfolio strategy. This idea will be repeated several times such that confidence intervals for the Sharpe Ratio can be determined.

3 Results

The previous section describes a way to model and analyze the information of the historical data. The model presented here is obviously a simplified version where only the overall trend is considered and the risk-free rate is a constant. The use of an overall trend means that only the global change of an asset value is considered, in a more detailed model the history can be split into different time intervals where a more local change or trend is maintained. It is important to note that in this report only one scenario is elaborated. The main assumption states that the assets categories will behave in the future as they did in the present.
Furthermore a distinction is made on the amount of historical data that is used. First, a simulation is performed based on the entire past. It is assumed that the economy follows a stationary regime even though it is well-known that the economic system can change over time. If a model based on the recent history is preferred then, secondly, one can take for instance only the last five years into account. This distinction is made in the following two subsections.

3.1 Situation 1: all historical data

In the figure below one simulation is represented for each of the four asset categories: cash, gold, bonds and stocks. The black line reflects the real historical data, the blue dotted line indicates the current time where the simulation of a possible future will start. The simulation itself is red. An extension of the global trend can be observed because all the information is included.

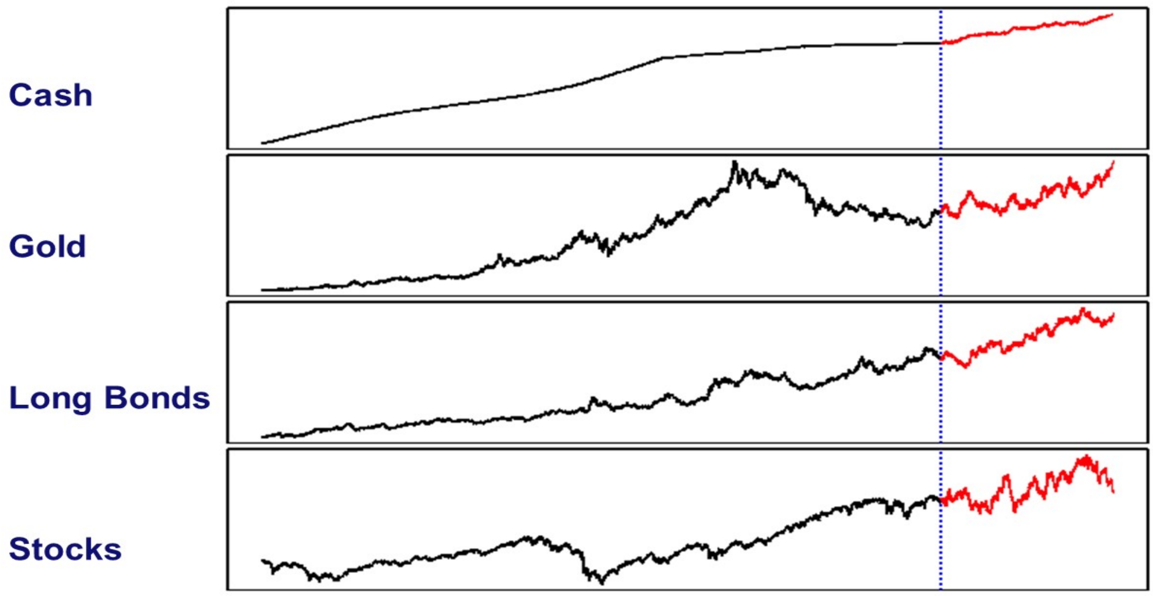


Figure 2: One simulation of the four asset categories: cash, gold, long bonds and stocks.

After the simulation of the separate asset categories, one is able to run a chosen strategy over the four classes. Here only the Permanent Portfolio is run over the data. For a simulation the resulting value is given in Figure 3. For this one simulation, the Sharpe Ratio can be calculated.

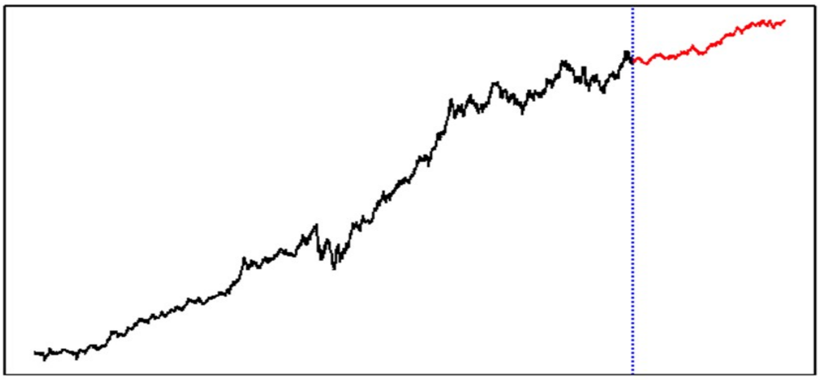
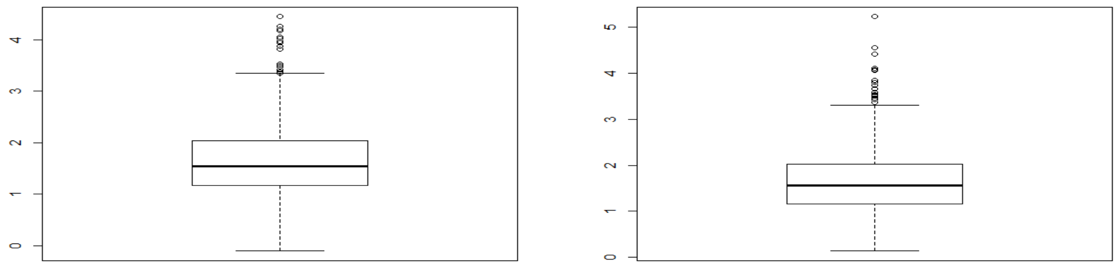


Figure 3: Value of the Permanent Portfolio for one simulation.

This process can be repeated several times, more specific the entire simulation was performed twice for one thousand separate possible futures and matching Permanent Portfolio values. For both cases, a boxplot of the corresponding Sharpe Ratios is given (Figure 4). These two boxplots are fairly similar, the expectation is that the more possible futures one simulates per case the more alike these boxplots will be.



(a) Boxplot 1a (b) Boxplot 1b

Figure 4: Boxplots Sharpe Ratios based one thousand simulation, situation 1: all historical data.

Figure 5 gives more global information of the value of the Permanent Portfolio. The mean trend of the performance is surrounded by 95% boundaries of the performance over all simuations runs, these boundaries are indicated in red.

Figure 5: Value of the Permanent Portfolio for one thousand simulation.

These results seem very promising, the median Sharpe ratio is above 1 what would indicate that the Permanent Portfolio is an excellent strategy. The Sharpe Ratio based only on the historical data is 0.78, which is much lower than the average values above. However the used model is a simplified version of the reality, the assumptions that were made are probably too loose. Furthermore, the future is uncertain which causes variation in the simulated results.

3.2 Situation 2: data of the five last years

Similar figures are given as in the subsection above, they only differ in the amount of data that is regarded. Figure 6 represents one simulation of the four asset categories: cash, gold, long bonds and stocks where only the last five years are modelled.

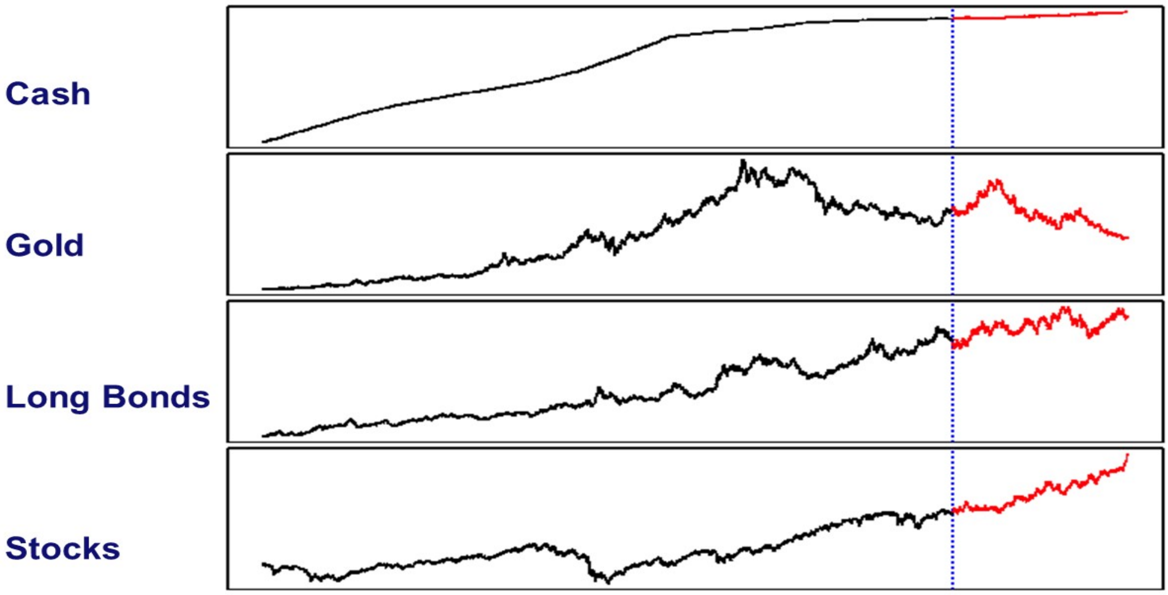


Figure 6: One simulation of the four asset categories: cash, gold, long bonds and stocks.

It can be observed that the growth of the economy the last five years is less steep compared to the overall growth of all the given data points. It can also be seen that for one simulation the growth ratio is smaller when only the last five years are considered, this is due to the assumption of repetition of the past. Figure 7 gives the Permanent Portfolio strategy run over the four asset categories of one simulation.

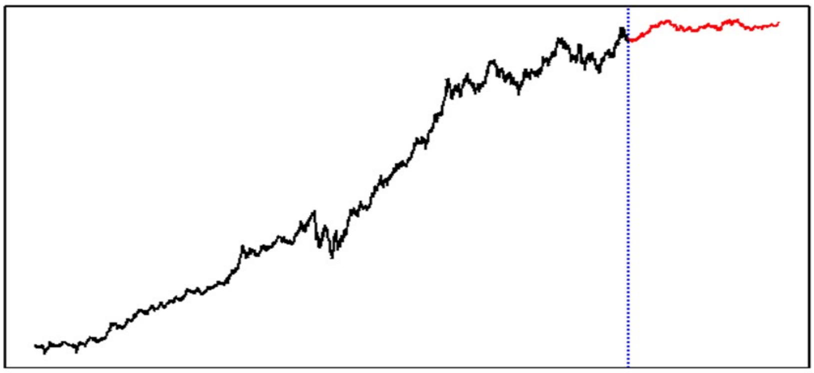
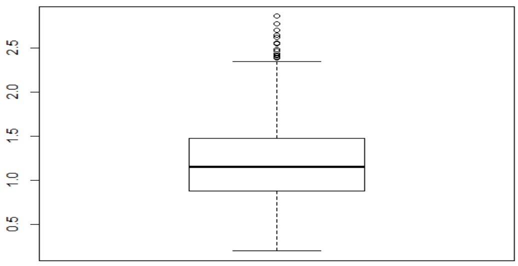
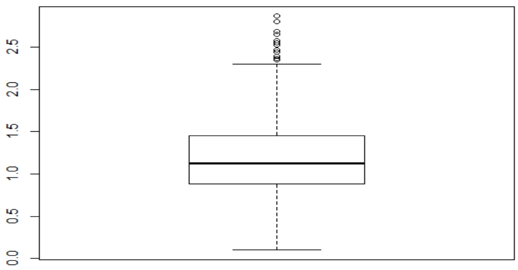


Figure 7: Value of the Permanent Portfolio for one simulation.

Similar as before, the procedure can be repeated one thousand times. For all these simulations the Sharpe Ratio can be calculated what will lead to the boxplots in Figure 8.



(a) Boxplot 2a



(b) Boxplot 2b

Figure 8: Boxplots Sharpe Ratios based one thousand simulation, situation 2: Data of the last 5 years.

The conclusion corresponding to these results is equivalent to the conclusion in the previous subsection.

4 Discussion

The first version of the assignment asked to compute a lower bound for the Sharpe ratio at 95% confidence interval level to obtain more certainty about the used strategy. Besides that it was asked how this strategy would perform in the future and to provide recommendations

on how to modify the optimization procedure such that it can be said with 95% certainty that the new strategy performs better than the base strategy in the future. This first question can mathematically not be answered as the Sharpe ratio is not a stochastic variable. Therefore the Sharpe ratio would just have a value that is either that value with 100% certainty or is not that value. To answer the questions sufficiently, the ideas were to model the data with the use of time series analysis and perform a Monte Carlo simulation based on the data provided by the model.
The results seem very promising as already mentioned. There are a few reasons for this. The used model is simplified by using the last known risk-free rate. This risk-free rate is very low at the moment, which causes the Sharpe Ratio to be higher than it would normally be. Another reason is that the used model the most simplified time series model is which is not suited for the gold asset. The final reason that these results seem promising is that it is based on past data, which does not guarantee the same results in the future. This leads to the ideas for further research. The first option is to improve the model, this can be done by modelling the risk-free rate as a stochastic value instead of a constant. The second improvement of the model can be to use a more advanced time series model than an AR(1) model. This improves the quality of the model. Besides changing the model, different strategies could be tested on a scenario which contains for instance a crisis or a totally different economic regime as they have in Japan. This can be used to verify whether the given strategy performs as well in difficult times as they would in economic good times.

5 Conclusion

To conclude, our project consists of three major parts. Firstly we have build a method that is able to construct an AR(1) model given historical data of a set of asset classes. The second part is a Monte-Carlo simulation that uses the model to predict many different futures. The final part is an implementation of the Permanent Portfolio strategy, which can calculate the value of a portfolio over time, given a possible future. By running Monte-Carlo simulations, we can find results about the performance of the Permanent Portfolio strategy. The main advantages of using a simulation is that we can find confidence statements, which is what we were originally looking for, and that multiple scenario's can be tested.
The main quantity we are interested in is the Sharpe ratio of a strategy. As such, our main result is the behaviour of the Sharpe ratio over the many possible futures. Though the average of the Sharpe ratio is quite good, it seems to have quite high variance, though this is to be expected since Sharpe ratio tends to stabilise after longer periods than the predicted range (however predicting longer ranges tends to decrease the accuracy of the model).
Important to note is that this method should not be used to predict what the future might bring. The simulation results should give a user an insight in how robust his strategy is. Modelling special events like a recent economic crisis or sudden abundance in oil, could be easily added to this model, and the simulation should tell the user something on how his strategy handles these events.
On a last note it should be mentioned that this AR(1) model is quite a simple one, which we chose considering the amount of implementation time at hand. However it can be considered as a proof of concept. Improving the model with more advanced technologies will most certainly result in better, more accurate results. We advise expanding this as a future improvement on our work.

As a conclusion, we constructed a useful tool for the customer to test and evaluate his strategies. Furthermore, there are plenty of possibilities to improve the proposed model.

References

[1] H. Browne. *Fail-Safe Investing: Lifelong Financial Security in 30 Minutes.* St. Martin's Griffin, 1st edition, 2001.

[2] D. Shumway, R. Stoffer. *Time Series Analysis and Its Applications.* Springer, 3rd edition, 2011.